

Computation of energy release rate for interfacial crack between dissimilar isotropic materials using mixed finite element

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Résumé:

Dans cet article, un élément fini mixte d'interface basé sur le principe variationnel mixte de Reissner est utilisé pour modéliser les interfaces fissurées entre deux matériaux. Cet élément d'interface a été associé à la méthode d'extension virtuelle de fissure pour évaluer le taux de restitution d'énergie. Les résultats obtenus, avec l'élément d'interface présenté, montrent une bonne concordance avec les solutions analytiques pour les biomatériaux isotropes.

Abstract:

In this paper, a mixed finite element, based on Reissner's mixed variational principle, has been used to model the cracked interfaces between two dissimilar materials. This interface element was associated with the virtual crack extension method to evaluate the energy release rate. Results obtained from the present mixed interface element have been shown to be in good agreement with the analytical solutions for isotropic biomaterials.

Keywords: Energy release rate, Virtual crack extension method, Mixed finite element, Cracked interface, Bimaterials.

1 Introduction

Fracture behavior of interface cracks between dissimilar materials is very important problem of composite materials. The problem of interfacial crack between two dissimilar isotropic materials was first investigated by Williams [1] who performed an asymptotic analysis of the elastic field at the tip of an open crack and found that the stress field possesses an oscillatory character [2]. Erdogan [3-4], England [5] and Rice and Sih [6] presented the asymptotic solutions of the stress field around the tip of a bimaterial interface crack.

In this paper, a mixed finite element presented by Bouziane and al. [7] has been used to model the cracked interfaces between two dissimilar materials. This interface element was associated with the virtual crack extension method to evaluate the energy release rate using only one meshing by finite elements. The accuracy of the element has been evaluated by comparing the numerical solution with an available analytical solution or numerical ones obtained from others finite elements. Results obtained from the present mixed interface element have been shown to be in good agreement with the analytical solutions.

2 Mixed finite element

The present element is a 7-node two dimensional mixed finite element: 5 displacement nodes with two degrees of freedom (two displacement components u_1, u_2) per node and 2 stress nodes with two degrees of freedom (two stress components σ_{22}, σ_{12}) per node. This element has been formulated starting from a parent element in a natural plane with an aim of modeling different types of interfaces with various orientations [7].

The final configuration of the element was obtained after passage by three following stages (figure 1):

1. Construction of a parent mixed finite element;
2. Relocalisation of certain variables inside the element and by displacement of static nodal unknown of the corners towards the side itself;
3. Static condensation of the internal unknown variables.

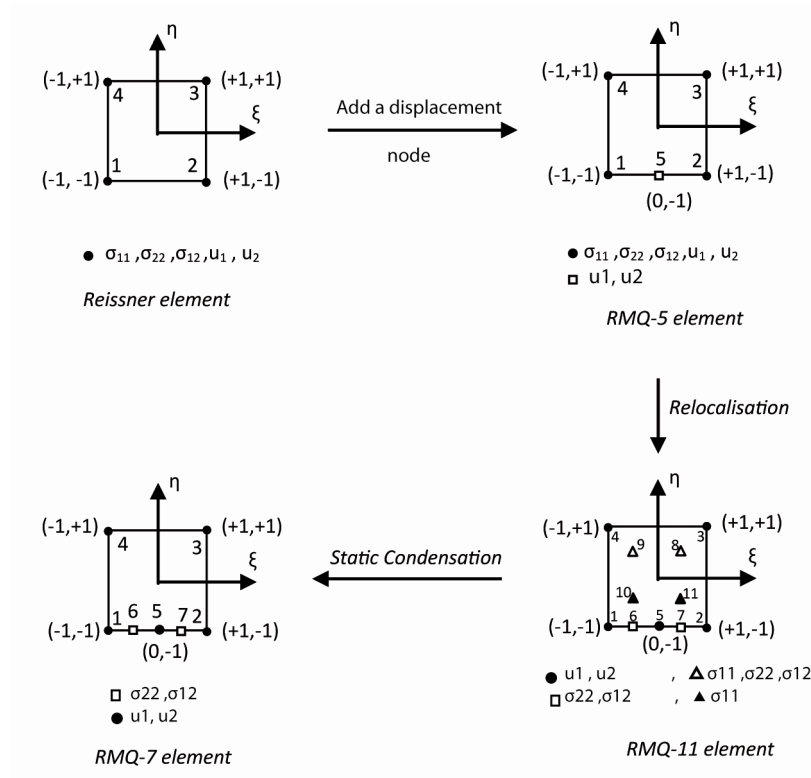


FIG. 1 – Stages of construction of the mixed finite element

3 Virtual crack extension method

The virtual crack extension method proposed by Parks [8] and Hellen [9] can calculate the energy release rate as:

$$G = -\frac{dU}{da} = -\frac{1}{2} \sum_{i=1}^{nc} \{u\}_i^t \frac{\Delta K_i}{\Delta a} \{u\}_i \quad (1)$$

where U is the potential energy of the system, a is the length of a crack, Δa is the length of the virtual crack extension, and ΔK_i and $\{u\}_i$ are the difference of the stiffness matrixes and the nodal displacements vectors of the elements i , surrounding a crack tip at the virtual crack extension, respectively. The evaluation of G by the virtual crack extension method requires two finite element analyses.

The use of the RMQ-7 (Reissner Modified Quadrilateral) element makes it possible to introduce only one mesh for the calculation of the energy release rate. The intermediate displacement node (node 5) of the RMQ-7 element is associated to the crack tip (figure 2), and consequently the length of crack a can be increased by a quantity Δa while acting inside strict of the crack element by translation of the node of crack tip without disturbing the remainder of the mesh [10].

In the used mesh, another equivalent element to that placed on the crack, is placed. This element has the same geometry and it is consisted of same material as show in figure 2.

The energy release rate is calculated starting from the difference of the elementary matrices of the element containing the crack representing the state $a+\Delta a$ and its equivalent element is representing the state a [10].

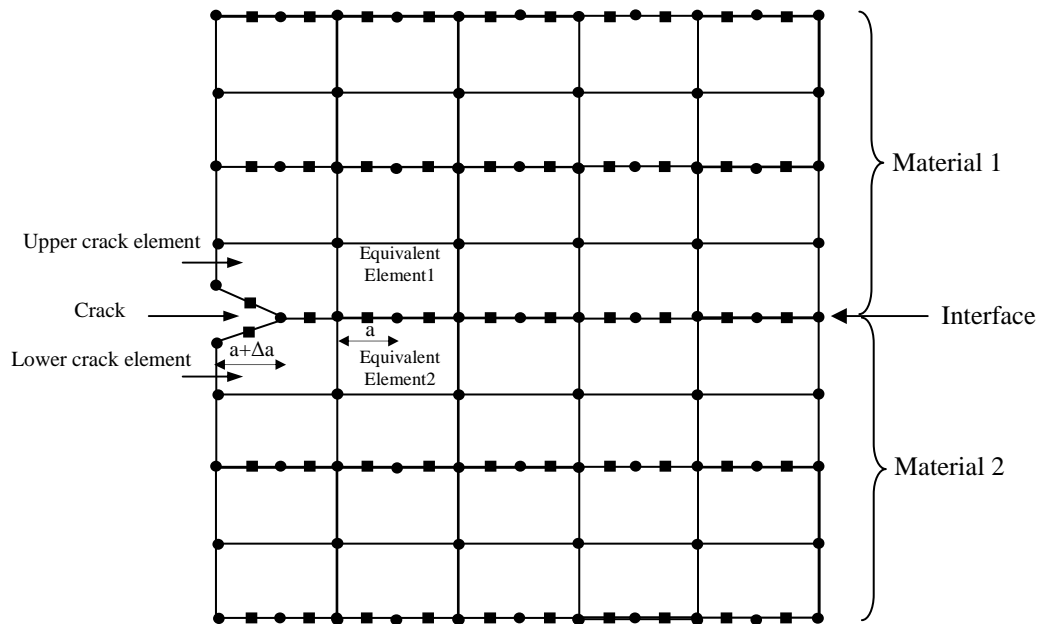


FIG. 2 - Mesh of cracked biomaterials

4 Numerical results and discussion

The computer program developed by incorporating the present mixed finite element (RMQ-7) has been employed for the analysis of a dissimilar square plate with a center crack in the interface plan between two isotropic materials [11] as shown in figure 3.

In this problem, the present element is associated to the virtual crack extension method to evaluate the energy release rate G . During numerical calculation, the choice of the crack length variation Δa is very important. To see the influence of this variation on the precision of calculation, we considered only one mesh with 50 elements and 286 degrees of freedom and we varied the extension in the interval $\Delta a / a = 1/10 \div 1/500$.

Results obtained with present interface element are compared with the values of the analytical solution [6] and the values of the numerical modelling of Lin and Mar [11]. These authors gave like results of their studies the stress intensity factors K_I and K_{II} from which we evaluated the energy release rate. Table 1 gives the values obtained according to E_1/E_2 .

E_1/E_2	Energy release rate $G(N/mm)$		
	Rice and Sih [6]	Lin and Mar [11]	Present mixed element
1	3.14	3.20	2.43
3	6.17	6.28	6.28
10	16.43	16.67	17.36
100	144.20	144.60	149.23

Table 1. Energy release rate of center interface crack between dissimilar materials

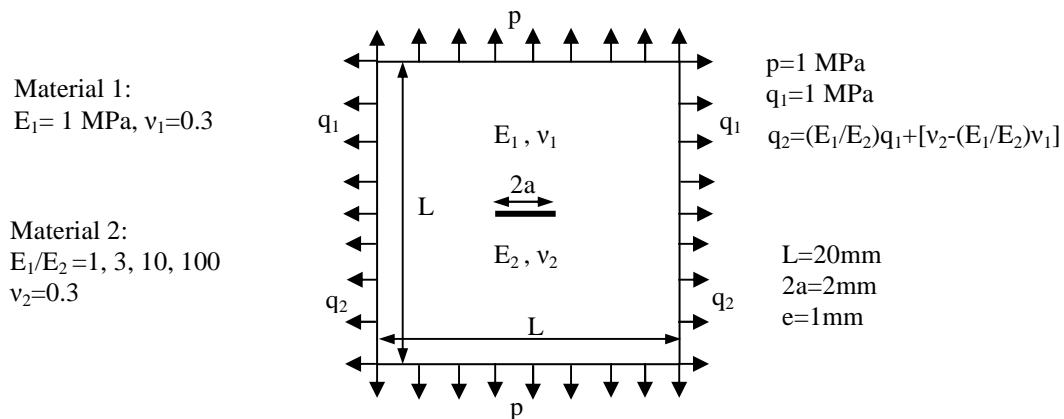


FIG. 3 - A center crack in a dissimilar square plate

The results obtained confirm the validation of the present element for the cracked structure. The choice of the length variation of crack Δa has a very significant role on the results precision. Indeed, it is necessary that this variation is sufficiently small so that the solutions obtained $u(a)$ and $u(a+\Delta a)$ are closer as much than the Δa extension is small compared to dimensions of the crack element. To highlight the importance of the choice of the extension Δa , we made numerical tests by using several values of $\Delta a/a$.

Table 2 gives the values of the energy release rate for various values of $\Delta a/a$ for $E_1/E_2 = 3$.

	$\frac{\Delta a}{a}$					
	$\frac{1}{50}$	$\frac{1}{100}$	$\frac{1}{150}$	$\frac{1}{200}$	$\frac{1}{250}$	$\frac{1}{300}$
Energy release rate G (N/mm)	3,68	4,15	4,67	5,20	5,71	6.28

Table 2: Energy release rate for various values of $\frac{\Delta a}{a}$

The results obtained confirm the importance which present the good choice of the crack variation length. We noted a very good stability between values $1/50$ and $1/300$ of the ratio $\Delta a/a$.

5 Conclusion

A special mixed finite element has been used to model cracked interfaces between two dissimilar materials. In the formulation of this element, we used Reissner's mixed variational principle to build the parent element. The mixed interface finite element is obtained by successively exploiting the technique of relocalisation and the static condensation procedure. This interface element was associated with the virtual crack extension method to evaluate the energy release rates using only one meshing by finite elements. The accuracy of the element has been evaluated by comparing the numerical solution with an available analytical solution or numerical ones obtained from others finite elements. Results obtained from the present mixed interface element have been shown to be in good agreement with the analytical solutions. Comparison of the results shows the validity of this mixed finite element for the treatment of the problems of interfacial crack between two dissimilar isotropic materials.

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